Worksheet for Section 7.2

Section 7.2 is about techniques for taking integrals involving trigonometric functions. Here are some problems that can be done using techniques you have already seen:

\[ \int \sin^3 x \cos x \, dx \quad \int \tan^5 x \sec^2 x \, dx \quad \int \sin^5 x \, dx \]

The last of the problems above is actually more like the rest of the problems in this section — the techniques in this section are not really new, but they are being applied in a new combination here. When dealing with integrals that have powers of sines and cosines, such as

\[ \int \cos^n x \sin^m x \, dx \]

There are some procedures you can follow to find what substitution to use and how to simplify the expression:

- if \( n \) is odd and positive, use \( u = \sin x \), so that \( du = \cos x \, dx \); use one \( \cos x \) in \( du \), convert the rest (now an even number) to sines using Pythagorean identity
- if \( m \) is odd and positive, use \( u = \cos x \), so that \( du = -\sin x \, dx \); use one \( \sin x \) in \( du \), convert the rest (now an even number) to cosines using Pythagorean identity
- if both \( n \) and \( m \) are even use the power reduction formulas below repeatedly until at least one power is odd

\[
\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}
\]

Using these procedures, work out the following integrals:

\[ \int \sin^5 x \cos^2 x \, dx \quad \int \sin^4 x \cos^7 x \, dx \quad \int \frac{\sin^3 x}{\sqrt{\cos x}} \, dx \quad \int \sin^4 x \, dx \]

Certain definite integrals of powers of sine and cosine work out nicely, and there are formulas for the values of the integrals, given on p. 492 as Wallis’s Formulas. These formulas are not particularly useful in practice, though, so we won’t be doing much with them.

There are similar procedures for working out integrals involving powers of the secant and tangent functions:

\[ \int \sec^n x \tan^m x \, dx \]

- if \( n \) is even and positive, use \( u = \tan x \), so that \( du = \sec^2 x \, dx \); use \( \sec^2 x \) in \( du \), convert the rest (still an even number) to tangents using Pythagorean identity
• if $m$ is odd and positive, use $u = \sec x$, so that $du = \sec x \tan x \, dx$; use
  $\sec x \tan x$ in $du$ (one of each), convert the rest of the tangents (now an even
  number) to secants using Pythagorean identity
• if $n = 0$ and $m$ is even, convert a $\tan^2 x$ to secants, expand, use the above
  rules, and repeat if necessary
• if $m = 0$ and $n$ is odd, use integration by parts
• if none of the above work, try converting to sines and cosines

Using these procedures, evaluate the following integrals:

\[
\int \sec^3 x \tan^3 x \, dx \quad \int \tan^4 x \, dx \quad \int \sec^4 x \tan^4 x \, dx \quad \int \sec^3 x \, dx
\]

There are also formulas on p. 410 for dealing with situations where you have
a product of sines and cosines of different angles, using product-to-sum identities.
Again, these don’t come up often, but it is worth looking at an example problem —
using those formulas, work out the following integral:

\[
\int \sin 4x \cos x \, dx
\]