Worksheet for Section 6.3

Section 6.3 is about a method for finding volume of solid regions using integrals: the Disk Method (with a variation called the Washer Method).

It is important first to understand solids of rotation — what happens when you take a region and rotate it around an axis to form a round solid. I will show you a picture in class to indicate what a solid of rotation is, and there are pictures in the book (see especially pp. 393–97).

The Disk Method is one way to find the volume of a solid of rotation. The important difference between the methods for finding volumes of solids of rotation is the orientation of the region being rotated, relative to the axis. In particular, if you would find the area of the region as an integral using approximating rectangles perpendicular to the axis, then you need to use the Disk Method.

\[ V = \int_a^b \pi y^2 \, dx = \int_a^b \pi [f(x)]^2 \, dx \]

For example, suppose the region under the curve \( y = x^2 \) on the interval \([0, 2]\) is rotated about the \( x \)-axis to generate a solid. What is the volume of this solid?

Sometimes with this method, the region does not come down to touch the axis of rotation — the Disk Method needs to be modified a bit to compute volumes in this case, since the disks would have circular holes in the middle. The resulting method is called the Washer Method (although it is really the same as the Disk Method). Here is the formula for volume computed this way:

\[ V = \int_a^b \pi \left( [f(x)]^2 - [g(x)]^2 \right) \, dx \]

\[ = \pi \int_a^b \left( [R(x)]^2 - [r(x)]^2 \right) \, dx \]

\[ = \pi \int_a^b [R(x)]^2 \, dx - \pi \int_a^b [r(x)]^2 \, dx \]

Here \( f(x) = R(x) \) is the outer radius of the washer and \( g(x) = r(x) \) is the inner radius (the radius of the hole in the middle). For example, suppose the region to be rotated is the region between the parabola \( y = x^2 \) and the line \( y = 2x \). Find the volume of the solid of rotation using the Washer Method, where the region is rotated about the \( x \)-axis. How would the computation change if the region were rotated about the \( y \)-axis? the horizontal line \( y = 5 \)?