

Worksheet for Section 12.7

Section 12.7 is about cylindrical and spherical coordinates, which provide coordinate systems in space akin to polar coordinates in the plane. Suppose P is a point in space. Most of this chapter has involved working with P using its rectangular coordinates (x, y, z) . The *cylindrical coordinates* for P take the form (r, θ, z) , where (r, θ) are the polar coordinate of (x, y) (equivalently, polar coordinates of the projection Q of P onto the xy -plane). So:

- to convert from cylindrical coordinates (r, θ, z) to rectangular coordinates (x, y, z) :

$$x = r \cos \theta \quad y = r \sin \theta \quad z = z$$

- to convert from rectangular coordinates (x, y, z) to cylindrical coordinates (r, θ, z) :

$$r = \sqrt{x^2 + y^2} \quad \tan \theta = \frac{y}{x} \quad z = z$$

I will show you in class a picture which gives a geometric interpretation of these formulas. For example, if P has cylindrical coordinates $(r, \theta, z) = (4, \frac{2\pi}{3}, 3)$, what are the rectangular coordinates for P ? If P has rectangular coordinates $(x, y, z) = (-3, -3, 5)$, what are the cylindrical coordinates for P ? Use the formulas above to write equations using cylindrical coordinates for the sphere $x^2 + y^2 + z^2 = 4$ and the plane $x + y - z = 4$.

The *spherical coordinates* for P take the form (ρ, θ, ϕ) , where ρ is the distance of P from the origin, θ is the angle in the xy -plane (as in cylindrical coordinates), and ϕ is the *angle of declination*, the angle between the line segment \overline{OP} and the positive z -axis. So:

- to convert from spherical coordinates (ρ, θ, ϕ) to rectangular coordinates (x, y, z) :

$$x = \rho \cos \theta \sin \phi \quad y = \rho \sin \theta \sin \phi \quad z = \rho \cos \phi$$

- to convert from rectangular coordinates (x, y, z) to cylindrical coordinates (r, θ, z) :

$$\rho = \sqrt{x^2 + y^2 + z^2} \quad \tan \theta = \frac{y}{x} \quad \cos \phi = \frac{z}{\rho}$$

I will again show you in class a picture which gives a geometric interpretation of these formulas. For example, if P has spherical coordinates $(\rho, \theta, \phi) = (2, \frac{2\pi}{3}, \frac{\pi}{4})$, what are the rectangular coordinates for P ? If P has rectangular coordinates $(x, y, z) = (3, -3, 3)$, what are the spherical coordinates for P ? Use the formulas above to write equations using cylindrical coordinates for the sphere $x^2 + y^2 + z^2 = 4$ — can you also write this in the form $\rho = f(\theta, \phi)$?