Worksheet for Section 12.5

Section 12.5 is about using vectors to find equations of planes in space. A plane in space will be specified using a point in the plane and a vector normal (i.e. perpendicular) to the plane. The plane containing point \( P = (x_1, y_1, z_1) \) with normal vector \( \vec{n} = (a, b, c) \) has equation in standard form:

\[
a(x - x_1) + b(y - y_1) + c(z - z_1) = 0
\]

By multiplying out and regrouping, the equation can be written in general form:

\[
ax + by + cz + d = 0
\]

Find the equation in standard form for the plane containing the points \( P = (1, 3, 2), \ Q = (-2, 2, 3), \) and \( R = (3, 0, -1) \). To do this, compute \( \vec{PQ} \times \vec{PR} \) — this will be the normal vector. What is the equation in general form for this plane? Sketch the points \( P, Q, \) and \( R \). Where is the plane relative to those points?

The angle between two planes, with normal vectors \( \vec{n}_1 \) and \( \vec{n}_2 \), is the same as the angle between the normal vectors:

\[
\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|}
\]

Find the angle between the plane you found above (containing the points \( P, Q, \) and \( R \)) and the \( xy \)-plane. (Hint: a normal vector for the \( xy \)-plane is one of the standard unit vectors.)

The distance between a plane (with normal vector \( \vec{n} \) and containing the point \( P \)) and a point \( Q \) (not in the plane) is:

\[
D = \|\text{proj}_{\vec{n}} \vec{PQ}\| = \frac{|\vec{PQ} \cdot \vec{n}|}{\|\vec{n}\|}
\]

If \( Q = (x_0, y_0, z_0) \) and the plane is given by the equation \( ax + by + cz + d = 0 \) (in general form), another formula can be used to find this distance:

\[
D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}
\]

Find the distance between the plane you found above (containing the points \( P, Q, \) and \( R \)) and the origin \((0, 0, 0)\).

The distance between a point \( Q \) and a line given by direction \( \vec{u} \) and point \( P \) is:

\[
D = \frac{\|\vec{PQ} \times \vec{u}\|}{\|\vec{u}\|}
\]

Find the distance between the point \( R = (3, 0, -1) \) and the line you found above, through the points \( P = (1, 1, 2) \) and \( Q = (-2, 2, 3) \). Is the distance between the origin and this line the same as the distance between the origin and the plane containing \( P, Q, \) and \( R \)?