Worksheet for Section 8.1

Section 8.1 is about how to compute the arc length of a graph and the surface area of a surface of rotation.

To get an idea of what the arc length of a curve represents, suppose you graph a function \( f(x) \) on an interval \( [a, b] \). Now take a piece of string and lay it down along the length of the graph of \( f \) that you drew. How much string do you need? This length (when measured in the same units you used for your graph) is the arc length of \( f(x) \). This length can be computed with the following formula:

\[
s = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx
\]

As a first example of how to use this formula, use it to verify the Distance formula for the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) — in this case, use the linear function \( f(x) = mx + b \); then what is \( f'(x) \), and how can you find this information from the coordinates of the points?

Here are some other examples — in each case, compute the arc length of the given function over the indicated interval:

- \( y = x^{3/2} \) on the interval \([0, 4]\)
- \( x = \frac{y^3}{6} + \frac{1}{2y} \) for \( y \) in the interval \([1/2, 2]\) (since this is a function of \( y \) instead of \( x \), use an integral with respect to \( y \) instead of \( x \))
- \( y = \ln(\cos x) \) on the interval \([0, \pi/4]\)

In Sections 6.2 and 6.3, you computed the volume of a solid of revolution, generated by rotating a region about an axis. In this section, the next topic is about computing the area of a surface of revolution — the idea for generating the object is similar, but now instead of rotating a region to get a solid, you only rotate a curve to get a surface, corresponding to the outer surface of the solid of rotation. (I will show you a picture in class.) The surface area of a surface of rotation can be computed using the following integral formula:

\[
S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2} \, dx
\]

(Notice the similarities between this formula and the arc length formula above.) For example, find the area of the surface of revolution generated by revolving the graph of \( y = x^3 \) on the interval \([0, 1]\) about the \( x \)-axis. Now do the same for the surface of revolution generated by revolving the graph of \( y = \sqrt{x} \) on the interval \([1, 4]\) about the \( x \)-axis.