Worksheet for Section 6.1

Section 6.1 is about computing the area between two curves, using definite integrals. Suppose the two curves between which you want to find the area are given by \( y = f(x) \) and \( y = g(x) \). Then the area between the curves is given by the following formula:

\[
\int_a^b f(x) - g(x) \, dx
\]

The limits \( a \) and \( b \) for this integral are often determined by where the curves intersect. For each of the pairs of functions given below, find the area between the curves:

- \( f(x) = 2 - x^2 \) and \( g(x) = x \)
- \( f(x) = x^3 \) and \( g(x) = 4x \)
- \( f(x) = 3x \) and \( g(x) = x^3 + 2x^3 \)

For each of these, you will need to start by finding out where the curves intersect. In the second and third examples, be careful — there are actually two regions in each of these for which you must find areas.

You can also do problems like this where the region is oriented horizontally rather than vertically. To do this, express the functions in terms of \( y \) rather than \( x \), then use the following formula:

\[
\int_c^d f(y) - g(y) \, dy
\]

(This is really the same formula as the first formula above, but in terms of \( y \) rather than \( x \).) Use this formula to find the area between the curves given by \( x = y - y^2 \) (a parabola) and \( x = 2y - 2 \) (a straight line).

There are some interesting applications of these formulas. We will look at a couple of word problems from the back of the section for examples.